ECON 1101-58 Fall 2004
ADDITIONAL NOTES #2

EXPLANATION OF CHAPTER 4

The material in Chapter 4 (excluding appendix) is about how to optimally choose the consumption level of one good. In order to describe this, the concept of marginal utility is developed and then the following argument is proposed: the optimal level of consumption of the good is the amount at which MU = P (marginal utility equals price). However, in the view of the material in the appendix this conclusion seems strangely paradoxical. Indeed, if we have a consumer with limited income and possibility to spend money only on the one good, consumer will just spend all his income in order to maximize his utility. This is of course very different from MU = P result obtained in the Chapter 4.

The question is: Why is this? Do we have two separate theories of consumer behavior? The correct answer is: NO. We have only one theory of consumer behavior and it is the one discussed in the appendix of Chapter 4.

In order to reconcile the material, make the following observation: According to our model of the consumer, money serve only as pieces of paper, which can let you buy goods that are available. If you don't spend money you don't get utility. However in Chapter 4 the discussion is about *monetary utility.* The difference is that in order to get the results of Chapter 4 we need to think of money as a separate good that brings us utility on its own. Moreover, the units of utility and money are the same: $1 = 1 util. As you can notice, this assumption is not that bad because it lets us analyze each good separately (think of the utility of money as just the utility of the stuff you can buy apart from the good under consideration).

Suppose we have two goods called *The Good* and *Money*. Suppose that the utility function of the consumer is $U(x) + y$, where x is the amount of The Good and y is the amount of Money. Suppose that consumer's income is I and suppose that the price of The Good is p (of course we also assume that the price of money is 1: you exchange money for money in proportion 1:1!). Therefore budget constraint of the consumer is

$$px + y = I$$

which is the same as

$$y = I - px$$

plug this into the expression for the utility and get: $U(x) + I - px$. Now we have only the amount of The Good in the formula, and this is what the consumer is maximizing!

Now consider marginal utility which is just the amount of additional utility one unit of The Good is bringing me ($MU(x) = U(x+1) - U(x)$ – plug different x and get the table as in the book). Notice that MU depends on the amount of good already at hand.

Now suppose that the consumer is trying to maximize $U(x) + I - px$ by choosing x. He is thinking if he should consume one more unit of x, thus he compares two utility levels: $U(x) + I - px$ and $U(x+1) + I - p(x+1) = U(x+1) + I - px - p$. If second number is bigger then obviously x is not optimal amount of consumption. This is to say

$$U(x+1) + I - px - p - (U(x) + I - px) = U(x+1) - U(x) - p > 0$$

or equivalently

$$MU(x) = U(x+1) - U(x) > p$$

So we conclude that if $MU(x) > p$ then x is not optimal level of consumption. In the same manner we can argue that if decreasing the consumption of the good increases the utility then x is not optimal level of consumption. Suppose the consumer consumes x units of the good and the utility of x-1 units is bigger:

$$U(x) + I - px - (U(x-1) + I - px + p) = U(x) - U(x-1) - p < 0$$

or

$$MU(x) = U(x) - U(x-1) < p$$

Notice I use the same notation $MU(x)$ for additional utility of *adding* one unit of the good or utility *lost* subtracting one unit. Therefore if $MU(x) < p$ then x is not optimal level of consumption. The only option left is $MU(x) = p$.

THE LAW OF DIMINISHING MU

Everything I said last time is consistent with this type of analysis. We can check that if our assumptions on the preferences hold then MU is decreasing. What are the assumptions?

- Given all other goods fixed, consumer always prefers more of any good to less (so this just says that MU > 0: adding one more unit of the good makes utility bigger – so the difference is positive)
- When the consumer has a very little amount of the good – he is willing to sacrifice *a lot of other stuff* to get more of that (this says that MU here is *big*: *other stuff* in our model is money, so this says that when consumer has little amount of the good he is willing to pay a lot for it, and since utility = money in this model, this translates to the high change in utility if additional unit of the good is consumed)
- When the consumer has too much of stuff – adding more of it does not change utility significantly (so here MU is *small*, adding one more unit of the good does not change the utility significantly – or the same: change of utility is small – or yet the same: consumer is not willing to pay much for additional unit of the good)

CONSUMER'S SURPLUS

When the consumer chooses his optimal level of consumption x, this means that he maximizes his utility equal to $U(x) + I - px$. So how much does he *gained* from the consumption of x units of the good (comparing to doing nothing and enjoying the income I)? The answer is simple: Before consumption, the consumer had his income I; Since money have utility in this model, this income gave the consumer utility I; the consumer spent all his income on buying x units of the good and this gave him the utility $U(x) + I - px$. The difference between these two number is $U(x) - px$. The difference between the utilities consumer gets before buying the goods to consume and after consumption is called *consumer's surplus*. This is the measure of how happy the consumer is after consumption. It is useful to learn how to show surplus graphically, this will help us a lot in the future analysis of the markets.

We can see surplus graphically on the graph of MU (demand). Suppose the price is p, then optimal x satisfies $MU(x) = p$. Now, *where on the graph is* $U(x) - px$?

We know $MU(x) = U(x) - U(x\text{-}1)$ and $U(0) = 0$ (no utility from zero amount of good), so

$$U(x) - px = U(x) - U(x-1) + U(x-1) - U(x-2) + U(x-2) - U(x-3) + \ldots + U(1) - U(0) - px =$$
$$= (MU(x) - p) + (MU(x-1) - p) + \ldots + (MU(1) - p)$$