

4109H: Game Theory

Midterm Exam

Instructor: Sasha Vostroknutov

November 1, 2005

**There are 6 pages and 100 points. You have 1 hour and 15 minutes.
Good luck!**

1. Consider the following normal form game:

	A	B	C	D
N	0 , 1	4.5 , 0	0 , 2	0 , 1
W	1 , 1	2 , 0	4 , 3	1 , 0
S	0 , 0	4 , 4	2 , 3.1	0 , 0
E	3 , 4	1 , 1	0 , 3	0 , 2

- a) (5 points) Find all pure Nash Equilibria of this game.

b) (10 points) Suppose this game is repeated two times. Each player's payoff is the sum of payoffs he gets in two stages. I claim that there is a NE in which none of the Nash Equilibria of the one-shot game is played in the first stage. Find such NE.

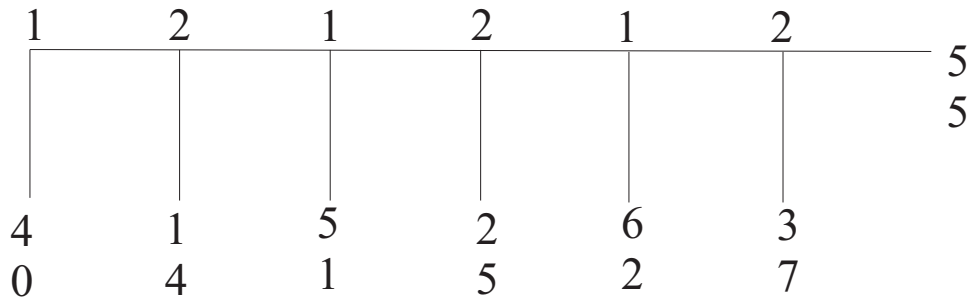
c) (10 points) Suppose now that both players discount second period with discount factor $\delta = 0.95$. So, their payoffs now are: payoff in stage 1 plus δ times payoff in stage 2. Is the NE you have found in part b) still NE? Prove your claim.

2. There are two firms on the market. The inverse demand is given by $P(Q) = a - Q$. If firm 1 produces q_1 it incurs the costs c_1q_1 and if firm 2 produces q_2 it incurs the costs c_2q_2 , where $c_1 > c_2 > 0$. Firm i 's ($i = 1, 2$) profit is therefore $P(q_1 + q_2)q_i - c_iq_i$
- a) (15 points) Suppose that firms simultaneously choose the quantities. Find a Nash Equilibrium.

b) (15 points) Now suppose that first firm 1 chooses its quantity, then firm 2 observes the choice of firm 1 and also chooses quantity. Find SPNE.

3. Consider the centipede game

(there are players 1 and 2, the top payoff is for player 1):



a) (10 points) Find SPNE: state the SPNE strategies (you can draw the arrows above).

b) (15 points) Suppose centipede game is repeated infinitely many times. For which δ can the outcome $(5, 5)$ be sustained as equilibrium payoff in each period? Find SPNE that does it and prove that it is SPNE (Hint: make sure your δ works for both players).

4. (20 points) Find all SPNE in pure strategies of the following game. State SPNE strategies (for each SPNE, make sure you specify who does what in *each* information set):