4109H: Game Theory
Some Math Problems

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1 Math Notation

$s \in S$ s is an element of (belongs to) set S;

$S = \{a, b, c, d\}$ set S consists of four elements;

$S = \times_{i=1}^{k} S_i$ or equivalently $S = S_1 \times S_2 \times ... \times S_k$. S is a Cartesian product of k sets. Typical element of S is $(s_1, s_2, ..., s_k) \in S$ such that $s_i \in S_i$ for all $i = 1..k$;

$(s_1, s_2, ..., s_k) \in S^k$ an element of Cartesian product $S^k = \times_{i=1}^{k} S$, so $s_i \in S$ for all $i = 1..k$;

$[0, 2) \subsetneq \mathbb{R}$ the interval from 0 (including) to 2 (not including) is a *strict* subset of real numbers, $\mathbb{R}$ (in general $T \subsetneq S$ means that T is a subset of S and $T \neq S$);

$T \subseteq S$ T is a subset of S. It might be true that $T = S$;

$u : S \to T$ u is a function from set S to set T, which means that for every element $s \in S$, u defines an element $u(s) \in T$;

$U : S \twoheadrightarrow T$ U is a correspondence from S to T. So, for every $s \in S$, U defines a non-empty subset $U(s) \subseteq T$;

2 Some problems

I designed these problems for you to have an idea of what type of math you should expect in this course. You will encounter problems like this all the time. These problems are **OPTIONAL**. However, in order to create an incentive, each person who submits these problems to me **NO LATER THAN 4PM on 12 of SEPTEMBER** will be able to add the points earned (maximum of 15) to the homeworks of your choice later in the course.

1. (5 points) In this exercise $u_i : S \to \mathbb{R}$ is some function for all $i \in I$. $s \in S$ is some fixed element of S, I is an arbitrary non-empty set. Negate the following statements:

    - For all $i \in I$ and for all $s' \in S$ $u_i(s) \geq u_i(s')$
    - For all $i \in I$ there exists $s' \in S$ such that $u_i(s) > u_i(s')$
    - There exists $s' \in S$ such that for all $i \in I$ $u_i(s) > u_i(s')$
    - There exists $i \in I$ and there exists $s' \in S$ such that $u_i(s) \leq u_i(s')$
    - There exists $i \in I$ such that for all $s' \in S$ $u_i(s) \leq u_i(s')$

2. (5 points) Solve the maximization problem below for all possible values of the parameter $p \in [0,1]$ (for each p find q's that attain maximum).

$$\max_{q \in [0,1]} 3qp - q(1-p) + 5(1-q)p - 2(1-q)(1-p)$$

    Write down a correspondence $U : [0,1] \twoheadrightarrow [0,1]$, which defines the solution to the above problem for any value of p ($U(p)$ is a subset of $[0,1]$ representing the solutions for that p). Draw a graph of U with p on the x-axis and q on the y-axis.

3. (5 points) Solve the maximization problem below for all possible values of the parameter $q \in [0,1]$ (for each q find p's that attain maximum).

$$\max_{p \in [0,1]} -(q+1)p^2 + 3p$$

Write down a correspondence $R : [0,1] \twoheadrightarrow [0,1]$, which defines the solution to the above problem for any value of q ($R(q)$ is a subset of $[0,1]$ representing the solutions for that q). Put the graph of R on the picture from exercise 2. Find all points (p^*, q^*) in which the two graphs intersect, or, formally, all (p^*, q^*) such that

$$q^* \in U(p^*)$$
$$p^* \in R(q^*)$$

SUGGESTION FOR 2 and 3: When solving maximization problems make sure you take boundaries for p and q into account. Also, make sure you find *all* solutions.