GAME THEORY 4109H: FINAL EXAM, FALL 2006

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THE FINAL IS DUE DECEMBER 12 AT 4PM IN CLASS. THERE WILL BE NO EXCEPTIONS OR LATE SUBMISSIONS.

THERE ARE 4 PROBLEMS AND 100 POINTS. THE FINAL SCORE IS 40% OF THE GRADE.

I expect clean, legibly written answers with detailed explanations of all reasoning that you do. Please, write down all equilibria you found in the end of each problem. All equilibria should be completely specified. For SPNE, I want to see what each player does in each information set. For PBNE, I want to see what each player does in each information set and what are the beliefs of each player in each non-trivial information set.

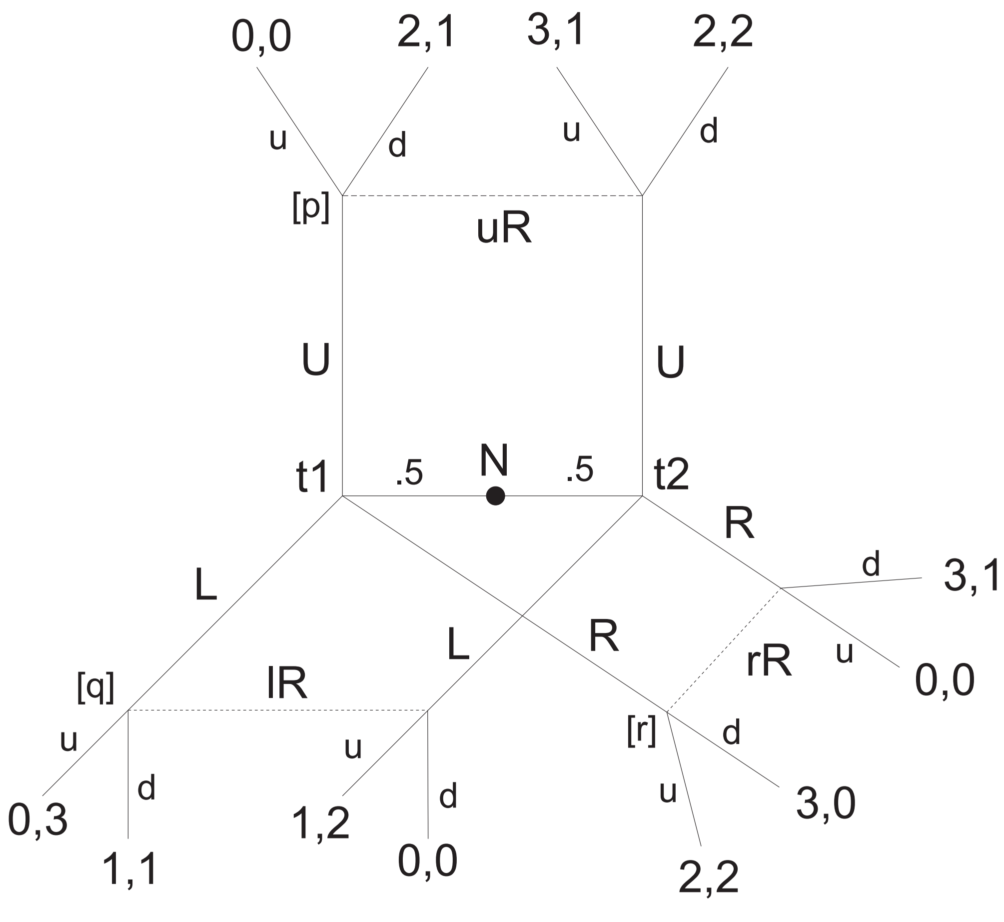
I WILL GRADE LESS LENIENTLY THAN HOMEWORKS. I WILL TAKE POINTS OFF FOR EACH NON-SPECIFIED EQUILIBRIUM AS WELL AS FOR EACH WRONG EQUILIBRIUM. I'M MOSTLY INTERESTED IN SEEING THE REASONING THAT YOU USE TO FIND EQUILIBRIA. YOU WILL GET MORE POINTS FOR CORRECT REASONING AND WRONG ANSWERS THAN FOR INCORRECT REASONING AND RIGHT ANSWERS.

GROUP WORK IS ALLOWED (SINCE I CANNOT PREVENT IT FROM HAPPENING ANYWAY). IF YOU HAVE ANY QUESTIONS, EMAIL ME TO SCHEDULE A MEETING.

Problem 1. Consider the signaling game below. There are two types of Sender: t_1 and t_2. Nature chooses either type with probability $\frac{1}{2}$. Each type can send a message to the Receiver. The message can be U, L, or R. Receiver observes a message and chooses an action u or d. The belief of the Receiver who sees message U (this receiver is called uR) is p. Analogously the beliefs of the Receiver seeing L and R are q and r correspondingly. The left payoff is of Sender the right payoff is of the Receiver.

1.a) **(15 points)** Find all PBNE. When stating your equilibria, please use the notation provided on the picture.

1.b) **(10 points)** Suppose now that Receiver does not observe the message of the Sender, so that the Sender and the Receiver choose their actions simultaneously. Right down this static game of incomplete information and find all BNE.



Problem 2. Consider four 2x2 games:

(s,s)	a	n
a	$(2.5, 2.5)$	$(1, 2)$
n	$(2, 1)$	$(3, 3)$

(s,w)	a	n
a	$(4, 1)$	$(1, 0)$
n	$(3, 2)$	$(0, 0)$

(w,s)	a	n
a	$(1, 4)$	$(2, 3)$
n	$(0, 1)$	$(0, 0)$

(w,w)	a	n
a	$(1, 1)$	$(3, 0)$
n	$(0, 3)$	$(5, 5)$

Player 1 chooses rows and Player 2 chooses columns. The action sets are $S_1 = S_2 = \{a, n\}$. Each player has two types: strong or weak. Nature puts probability p on the strong type of each player. The draws are independent across players. One of the four 2x2 games is played depending on the types of the players (upper left corner of each game): for example (w, s) means that Player 1 is weak and Player 2 is strong.

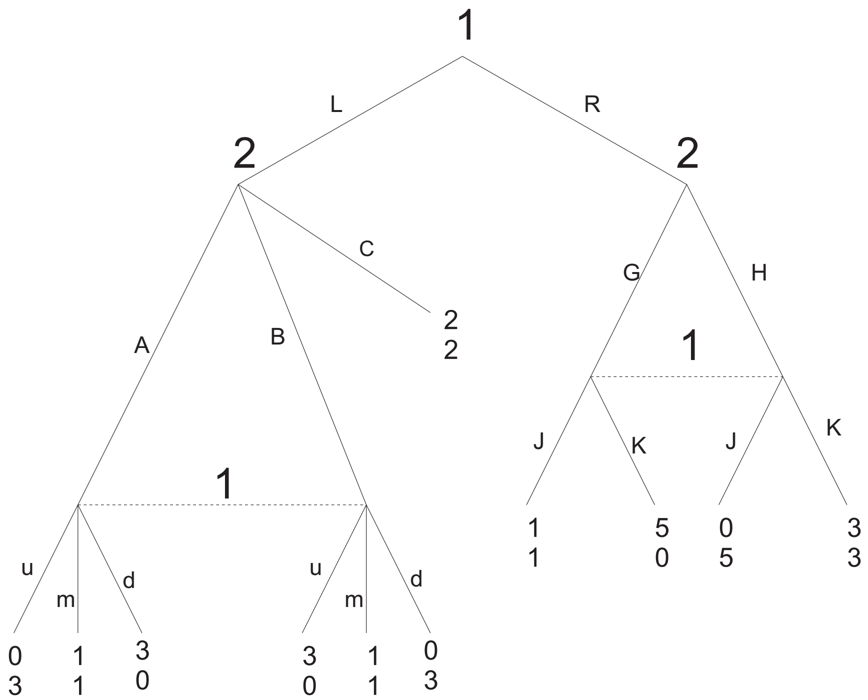
2.a) **(10 points)** Suppose that before the game is played, Nature informs each player *only* about that player's type, but not about the type of the other player. What are the type sets for both players? Find all pure strategy BNE of the resulting game for all possible $p \in [0, 1]$.

2.b) **(10 points)** Now suppose that before the game is played Nature informs Player 1 about both her type and the type of Player 2; Player 2 knows her type, *but does not know* the type of Player 1. What are the type sets for both players? Find all pure strategy BNE of the resulting game for all possible $p \in [0, 1]$.

2.c) **(10 points)** Now assume that Nature tells both players their own types and the type of the other player, so both Player 1 and Player 2 know everything about the types. What are the type sets for both players? Find all pure strategy BNE of the resulting game for all possible $p \in [0, 1]$.

Problem 3. (20 points) Do Gibbons 3.6.

Problem 4. Consider the game below



4.a) **(5 points)** Find pure strategy SPNE.

4.b) **(10 points)** Can the SPNE you found above be sustained as PBNE? If yes, find this PBNE. If not, explain why.

4.c) **(10 points)** Suppose we allow Player 1 to choose actions probabilistically (Player 1 can choose u, m and d with probabilities γ_1, γ_2 and $1 - \gamma_1 - \gamma_2$). Find a *mixed strategy* PBNE.