

4109H: Game Theory Midterm Exam. Instructor: Sasha Vostroknutov

There are 7 pages. You have 1 hour and 15 minutes.

THERE ARE 100 + 10 POINTS. GOOD LUCK!

1. Consider the following normal form game:

	A	B	C
N	2 , 1	1 , 0	0 , 4
W	0 , 3	4 , 1	0 , 2
S	3 , 4	2 , 2	1 , 3
E	0 , 3	0 , 4	4 , -1

- a) (10 points) Perform Iterated Elimination of Strictly Dominated Actions until you cannot proceed further (state the order in which you eliminate the actions)

b) (10 points) Find all pure and mixed Nash Equilibria of this game

c) (10 points) Suppose the game is repeated 2 times. Are there SPNE in which non-NE outcome is played in the first stage? If yes, give an example of such SPNE. If no, explain why.

2. The economy consists of two firms called 1 and 2 and two goods called G_1 and G_2 . Firm i produces only good G_i . The goods are substitutes, so the inverse demand for the good G_i , $i = 1, 2$ is given by $P_i(q_1, q_2) = a - q_i - kq_{-i}$, where q_i is the amount of good G_i produced by firm i and $1 \leq k < 2$ is a constant. The costs for both firms are cq_i . Firm i 's ($i = 1, 2$) profit is therefore $P_i(q_1, q_2)q_i - cq_i$.
- a) (15 points) Suppose that firms simultaneously choose quantities. Find a Nash Equilibrium.

b) (15 points) Now suppose that first firm 1 chooses its quantity, then firm 2 observes the choice of firm 1 and also chooses quantity. Find SPNE.

3. For any $0 < x < 2.5$ consider the following game:

	C	D
C	x, x	$5, 0$
D	$0, 5$	$0, 0$

- a) (20 points) Suppose that this game is repeated infinitely many times. I claim that for discounting factor δ high enough there is a NE of infinitely repeated game in which players agree to alternate between the outcomes (D, C) and (C, D) forever. So the equilibrium path is $(D, C), (C, D), (D, C), (C, D) \dots$. Describe the strategies of the two players that implement this NE. Find for which δ this NE works (depending on x of course).

b) (10 points) Is the NE you have just constructed also a SPNE? Prove your claim rigorously.

4. (20 points) Find all SPNE in pure strategies of the following game. State SPNE strategies (for each SPNE, make sure you specify who does what in *each* information set):