4109H: HOMEWORK 4 (due November 16 in class)

1) (25 points) Gibbons 3.2

2) (15 points) Gibbons 3.4

3) (25 points) Simple Bertrand Competition. Two firms play Bertrand by choosing Low, Medium or High price. Suppose that firm 1, which chooses rows, does not know if the good it produces is a substitute or a compliment of firm 2's good. The probability of firm1's good being substitute is $p \in [0, 1]$. Firm 2 knows if the goods are substitutes or compliments. The payoffs are depicted in the matrices:

Substitutes	High	Medium	Low	**Compliments**	High	Medium	Low
High	5 , 5	0 , 8	0 , 6	High	5 , 5	6 , 3	10 , 1
Medium	8 , 0	4 , 4	0 , 6	Medium	3 , 6	4 , 4	5 , 2
Low	6 , 0	6 , 3	3 , 3	Low	1 , 10	2 , 5	3 , 3

For each value of p find *some* pure-strategy BNE, if for some values of p there are no pure strategy BNEs, prove it **(HINT: Look for strictly dominated actions that can be eliminated. Beware though! The firm that has no information about which game is played MIGHT play strictly dominated action!)**.

4) (35 points) Consider the following strategic situation. Two opposed armies are poised to seize an island. Each army's general can choose to either "attack" or "not attack". In addition, each army is either "strong" or "weak" with equal probability (the draws for each army are independent), and an army's type is known only to its general. Payoffs are as follows: The island is worth M if captured. An army can capture the island in one of the two cases: 1) by attacking and its opponent does not attack; 2) by attacking if it is strong and its rival also attacks but rival is weak. If two armies of equal strength both attack, neither captures the island. An army also has a cost of fighting, which is s if it is strong and w if it is weak, where $s < w$. There is not cost of attacking if the rival does not. **Suppose that $M - 2s > 0$ and $M - w < 0$.**

a) Write the normal form of this game and identify all pure strategy BNE.