4113: Math Econ Final

DUE WEDNESDAY MAY 10, 4:30PM AT HELLER HALL 1035.
THE 1035 OFFICE CLOSES AT 4:30PM - SO DON'T BE LATE!

1. (60 points) Solve the following problem:

$$\begin{aligned} \max_{x,y,z\in\mathbb{R}} \quad & (x+y)^2 + 5xy + x - y \\ \text{s.t.} \quad & x^2 + y^2 \le z^2 \\ & z \le 1 \\ & x^2 - \frac{1}{4} \le y \\ & x, y, z \ge 0 \end{aligned}$$

   This problem does NOT have simple solution, in addition the numbers are not neat either (like in real life!). If you use Lagrange theorem DO CHECK CONSTRAINT QUALIFICATION. You don't have to find exact numerical solution. It is enough to give an equation in one variable (say y) and say that the solution to this equation is the optimal y. Also specify how to find optimal x and z in terms of that optimal y. You might find it useful to use the theorems about the existence of a solution, etc. that I gave in class. You also should be able to simplify the problem a bit by getting rid of some inequalities (prove that you can do that). Try to draw a graph in $x - y$ plane and see what kind of function you are maximizing and over which set.

2. (40 points) Consider the following intertemporal optimization problem:

$$\begin{aligned} \max_{\{c_t,k_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t c_t \\ \text{s.t.} \quad & k_{t+1} + c_t \le \sqrt{k_t} \\ & c_t, k_t \ge 0 \\ & k_0 \quad \text{given.} \end{aligned}$$

   a) Formulate this problem as a dynamic program: denote by $V(k)$ the utility the consumer gets if he starts from capital k and chooses all variables optimally in the future.

   b) In part a) you obtained the functional equation, with unknown function V. Find the explicit solution to this equation using the "guess and verify" method. Hint: assume that $V(k) = Af(k) + B$, where A and B are some constants and f - some function.

   c) In part b) in order to find $V(k)$ you should have found the optimal next period capital k' in terms of A, B and k. Use this information to derive optimal path of consumption $\{c_t^*\}_{t=0}^{\infty}$ and capital $\{k_t^*\}_{t=0}^{\infty}$. Assume that $(\frac{\beta}{2})^4 < k_0 < 1$.