

ECON 4113. HOMEWORK 1 (100 POINTS). DUE THURSDAY JANUARY 26 IN CLASS.

For all of the following problems I would like to see your work, not just answers. You should be able to solve everything below if you read Simon and Blume. For your reference: very useful web-sites for finding definitions of mathematical terms:

<http://planetmath.org/>

<http://mathworld.wolfram.com/>

1. (15 points) Consider a function $(x, y, z) \mapsto (x^2 + 2y^2 + 3z^2)e^{-(x^2+y^2+z^2)}$ where $x, y, z \in \mathbb{R}$ (here is another useful piece of notation: people use the symbol \mapsto when they don't want to give the function a name). Find all critical points (those where Jacobian derivative is 0).
2. (10 points) Find the determinant and the rank of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 1 & 0 & 8 \end{pmatrix}$$

3. (15 points) Find out if the following matrix is positive definite, negative definite or indefinite:

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{pmatrix}$$

4. (25 points) Let $F : \mathbb{R}^3 \rightarrow \mathbb{R}$ be given by $F(x, y, z) = xye^{-z}$. Find the Jacobian derivative J of F . If you think of J as a function, what are its domain and target spaces?
5. (35 points) Find the Jacobian derivative H of J from the previous exercise. If you think of H as a function, what are its domain and target spaces? How are H and F related? Find all critical points of F and determine which of them are local maxima, local minima and saddle points.