ECON 4113. HOMEWORK 3 (100 POINTS).
DUE THURSDAY MARCH 2 IN CLASS.

1. (20 points) Dixit, problem 6.3, page 85. Don't forget to assume $x_1, x_2 \geq 0$.

2. (25 points) Simon & Blume, problem 21.19, page 527. You are asked to prove that three statements are equivalent. Statement a is equivalent to statement b whenever it is true that $a \Rightarrow b$ and $b \Rightarrow a$. To show the equivalence of three statements a, b and c it is enough to show that

$$a \Rightarrow b \Rightarrow c \Rightarrow a$$

   or any other chain of the same type (like b, c, a, b instead of a, b, c, a). When proving the Theorem, you should think first which implications are easy to prove and use them as the part of the chain. Hint: read Dixit, Chapter 6.

3. (15 points) Suppose there are two goods x_1 and x_2. The consumption set is $\mathbb{R}^2_{++}$. We have one consumer with lexicographic preferences, who prefers any infinitesimal increase of x_1 to any amount of x_2. For any price system (p_1, p_2) find the optimal consumption bundle.

4. (20 points) Revealed Preference Exercise. Set up is the same as in the previous problem. Suppose that you are given the data about the bundles that some consumer is choosing given price system (p_1, p_2). Namely, for any (p_1, p_2) you know which bundle was chosen. Suppose that these data coincides with what you've found in the previous exercise. Can you conclude that this consumer has lexicographic preferences? Is there a preference relation on $\mathbb{R}^2_{++}$ that is not lexicographic, has a utility function and generates exactly the same data points you observe?

5. (10 points) Suppose $\succsim$ is transitive and irreflexive. Prove that it is asymmetric. Hint: use proof by contradiction. (Negation of $(a \Rightarrow b)$) is equivalent to (a and not b):

$$\neg(a \Rightarrow b) \text{ is equivalent to } (a \wedge \neg b)$$

6. (10 points) Suppose $\succsim$ is transitive and asymmetric. Prove that it is irreflexive.