ECON 4113. OPTIONAL HOMEWORK PART 1. DUE APRIL 27.

Solve the following problems by all means necessary. If you use Lagrange Theorem do the Constraint Qualification properly. There is no need to check second order conditions.

1.

$$\begin{aligned} \max_{x,y,z\in\mathbb{R}} \quad & zx + y^2 \\ \text{s.t.} \quad & (y^2 - 1 + 2zx)^4 \leq 0 \\ & x, y, z \geq 0 \end{aligned}$$

2.

$$\begin{aligned} \max_{x,y\in\mathbb{R}} \quad & 9y + 2xy - 2x^2 - 2y^2 \\ \text{s.t.} \quad & 4x + 3y \leq 10 \\ & y - 4x^2 \geq -2 \\ & x, y \geq 0 \end{aligned}$$

3.

$$\begin{aligned} \max_{x,y\in\mathbb{R}} \quad & x \\ \text{s.t.} \quad & y - x^4 \leq 0 \\ & x^3 - y \leq 0 \\ & x \leq \frac{1}{2} \end{aligned}$$

4. Consider the problem of maximizing the utility of a consumer in an exchange economy, keeping the utility of the other consumer no less than some fixed level $\bar{u}_2$:

$$\begin{aligned} \max_{x_1,y_1,x_2,y_2\in\mathbb{R}} \quad & u_1(x_1, y_1) \\ \text{s.t.} \quad & u_2(x_2, y_2) \geq \bar{u}_2 \\ & x_1 + x_2 \leq w_x \\ & y_1 + y_2 \leq w_y \\ & x_1, y_1, x_2, y_2 \geq 0 \end{aligned}$$

Suppose that u_1 and u_2 are strictly increasing in all arguments. Show that at the solution point $(x_1^*, y_1^*, x_2^*, y_2^*)$ the first three constraints bind. Suppose for simplicity that $x_1^*, y_1^*, x_2^*, y_2^* > 0$. Find the conditions that $x_1^*, y_1^*, x_2^*, y_2^*$ should satisfy.