1 Preference Relations

Suppose we have a binary relation R (you can use any other symbol, for example $\succsim$) on the set of alternatives X. Remember that $R \subseteq X^2$ and for $x, y \in X$ we say that xRy is true if $(x, y) \in R$. Here are some definitions. You should read the statements below as follows: R is **reflexive** if for all $x \in X$ it is true that xRx.

reflexive if xRx

irreflexive if $\neg(xRx)$

transitive if xRy *and* $yRz \Rightarrow xRz$

total if $x \neq y \Rightarrow [xRy$ *or* $yRx]$

complete if xRy *or* yRx *or both*

symmetric if $xRy \Rightarrow yRx$

asymmetric if $xRy \Rightarrow \neg yRx$

To understand these definitions think of two preference relations " $=$ " and " $>$ " defined on the real line. Consider $=$ first: it is reflexive ($x = x$ is true for any $x \in \mathbb{R}$); transitive ($x = y = z$ implies $x = z$); symmetric ($x = y$ implies $y = x$); not total ($x \neq y$ does not imply $x = y$ or $y = x$). Now think of $>$: it is irreflexive ($\neg(x > x)$ is true); transitive ($x > y > z$ implies $x > z$); total ($x \neq y$ implies either $x > y$ or $y > x$); asymmetric ($x > y$ implies not $y > x$).