

4113: Math Econ Final
DUE MONDAY MAY 7, 3:00PM AT HELLER HALL 1035.
DON'T BE LATE. I WILL GET THE FINALS AT 3:15PM.
THERE WILL BE ABSOLUTELY NO WAY
YOU CAN GIVE ME THE FINAL LATER THAN THAT
SINCE I WILL LEAVE MINNEAPOLIS.

1. (30 points, ECON4113 FINAL PROBLEM) Solve the following problem:

$$\begin{aligned} \min_{x,y,z \in \mathbb{R}} \quad & e^x + e^{-y} + z \\ \text{s.t.} \quad & e^x \leq 1 \\ & e^{-y} \leq 1 \\ & (x-1)^2 + y^2 \leq 1 \\ & x^2 + y^2 + e^{-z} \leq 1 \end{aligned}$$

Notice it is a MINIMIZATION problem. If you use Lagrange theorem DO CHECK CONSTRAINT QUALIFICATION.

2. (30 points, ECON4113 FINAL PROBLEM) Solve the following problem

$$\begin{aligned} \min_{x,y \in \mathbb{R}} \quad & (x+1)^2 + (y+1)^2 \\ \text{s.t.} \quad & y \geq x^4 + 1 \\ & y \leq x^3 + 1 \end{aligned}$$

Notice it is a MINIMIZATION problem. If you use Lagrange theorem DO CHECK CONSTRAINT QUALIFICATION.

3. (40 points, ECON4113 FINAL PROBLEM) Consider the following intertemporal optimization problem:

$$\begin{aligned} \max_{\{c_t, k_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t c_t \\ \text{s.t.} \quad & k_{t+1} + c_t \leq \sqrt{k_t} \\ & c_t, k_t \geq 0 \\ & k_0 \text{ given.} \end{aligned}$$

- a) Formulate this problem as a dynamic program: denote by $V(k)$ the utility the consumer gets if he starts from capital k and chooses all variables optimally in the future.

- b) In part a) you obtained the functional equation, with unknown function V . Find the explicit solution to this equation using the “guess and verify” method. Hint: assume that $V(k) = Af(k) + B$, where A and B are some constants and f - some function.
- c) In part b) in order to find $V(k)$ you should have found the optimal next period capital k' in terms of A , B and k . Use this information to derive optimal path of consumption $\{c_t^*\}_{t=0}^\infty$ and capital $\{k_t^*\}_{t=0}^\infty$. Assume that $(\frac{\beta}{2})^4 < k_0 < 1$.