

**ECON 4113. HOMEWORK 0 (100 POINTS).
DUE THURSDAY FEBRUARY 1 IN CLASS.**

For all of the following problems I would like to see your work, not just answers. HW written by hand is fine as long as it is legible. For your reference: very useful web-sites for finding definitions of mathematical terms:

<http://planetmath.org/>

<http://mathworld.wolfram.com/>

1. (20 points) Make truth tables for each of the following:

a. $P \wedge (Q \vee \sim Q)$

b. $(P \wedge Q) \vee \sim Q$

c. $(P \vee Q) \Rightarrow (P \wedge Q)$

d. $((\sim P) \Rightarrow Q) \vee (Q \Leftrightarrow P)$

2. (20 points) Translate the following English sentences into symbolic sentences with connectives (\vee, \wedge, \sim) and quantifiers (\forall, \exists). For example: “No one loves everybody (universe: all people)” is

$$\sim [\exists x \forall y (x \text{ loves } y)]$$

a. All precious stones are not beautiful (universe: all stones)

b. All people are honest or no one is honest (universe: all people)

c. There is a smallest positive integer (universe: integers)

d. Between any real number and any larger real number there is a rational number (universe: real numbers)

3. (30 points) Prove or disprove the following statements (write clear proofs with steps, specifying which type of proof you are using: direct, contrapositive, contradiction etc.). Before starting the proof specify if you are proving the statement or disproving it (proving the negation of it).

a. $\forall x \in \mathbb{R} \quad x > 0 \Rightarrow x^2 - x > 0$

b. There is no smallest positive real number

c. $\forall \varepsilon > 0 \quad \exists M \in \mathbb{N} \quad \forall n > M \quad \frac{1}{n} < \varepsilon \quad (\varepsilon \text{ is real, } n \text{ is natural number})$

4. (30 points) Consider real line \mathbb{R} . For any set $A \subseteq \mathbb{R}$ the compliment of the set is $A^c = \mathbb{R} \setminus A$ (everything that is not in A). Prove the following statement: If A is closed then its compliment A^c is open [Hint: use definitions of open and closed given in class].