**ECON 4113. HOMEWORK 4. 100 POINTS. DUE APRIL 17. OFFICE HOURS - THURSDAYS, TUESDAYS, BEFORE-AFTER CLASS.**

1. (20 points) Consider a modification of a search problem discussed in class. A worker lives for infinitely many periods. In each period she can do two things if she is unemployed: 1) accept a wage $w$ that comes from some known distribution $\Phi$; 2) reject the wage. In case 1, the worker gets wage $w$ in current period and in the next period she can also get wage $w$ if still employed or she can get unemployed and have a draw from the distribution $\Phi$ again. The worker in the next period can get unemployed with probability $\gamma$. In case 2, she gets 0 and in the next period she searches again. If worker is employed she does nothing. The discount factor is $\delta$. Formulate this problem as a dynamic program (as in class). Show that the solution to this program, the function $V(w)$, has the reservation wage form. Namely, if the wage draw is lower than some $w^*$, the worker rejects and if it is higher, she accepts. Do NOT solve for $w^*$.

2. (40 points) Consider the modification of the Red-n-Black problem solved in class. We have an agent who lives for $n$ periods. In period 1 he is endowed with $x$ dollars. Each period he decides on a bet $a$, where $0 \le a \le x$. In the next period, depending on the realization of a coin flip he gets $x + a$ with probability $w > \frac{1}{2}$ or $x - a$ with probability $1 - w$. After this the betting repeats. The utility, the agent gets, is calculated in the end of the game. If in period $n$ (in which no betting is happening, since it is the last period) the agent has $x_n$ dollars then his utility is $u(x_n) = \ln x_n$. The agent (being in period 1) tries to solve the problem

$$\max E[u(X_n)]$$

where $X_n$ is a random variable generated by the coin flips together with the decisions the agent makes on the way. Find the optimal betting strategy, using the same technique as in class.

3. (40 points) Consider the 3 period "Eat-or-Save" problem as discussed in class. In period 0 a consumer is given some capital $k_0 > 0$. He can consume it or save for the later periods. Consumer solves the following problem:

$$\begin{aligned} \max_{c_{0,1,2};k_{1,2}\in\mathbb{R}} \quad & \sum_{t=0}^{2} \beta^t \ln c_t \\ \text{s.t.} \quad & k_{t+1} + c_t \le k_t, \qquad t = 1, 2 \\ & c_2 \le k_2 \\ & c_t, k_t \ge 0, \qquad t = 0, 1, 2 \end{aligned}$$

Formulate this problem as a sequence of one period maximization problems, using the technique shown in class. Find the optimal values $V_0^*, V_1^*, V_2^*$ and optimal paths of capital and consumption $\{c_0^*, c_1^*, c_2^*, k_0^*, k_1^*, k_2^*\}$. If math becomes too messy, just express these variables in terms of some other, which are known.