ECON 4113. OPTIONAL HOMEWORK 5. 100 POINTS. DUE APRIL 24.

All you need to solve this homework is in Chapters 24 and 25 of Simon and Blume. All terms they refer to (like eigenvalues) are defined in some other chapters of Simon and Blume (use index in the end to locate it). To determine stability properties in problems 2 and 3 use Theorems 25.1, 25.2 and 25.3 of S& B.

1. (35 points) Draw the direction fields (t on the x axis) for the following differential equations:

   a) $\dot{y} = \frac{ty}{1+t^2}$

   b) $\dot{y} = y(y-2)$

   c) $\dot{y} = t^2 - y^2$

2. (40 points) Consider a Cournot duopoly with heterogeneous firms. Suppose that the inverse demand is given by $P(Q) = a - Q$. Firm 1 has cost c_1 to produce one unit of output and firm 2 has cost $c_2 \neq c_1$.

   a) Find the unique Nash Equilibrium of the Cournot duopoly game. Under which conditions on c_1 and c_2 do both firms produce? Under which conditions on c_1 and c_2 does only one firm produce?

   b) Suppose the firms change the quantities q_1 and q_2 at each time $t \in [0, \infty)$. Consider the following dynamics:

   $$\begin{aligned} \dot{q}_1 &= BR_1(q_2) - q_1 \\ \dot{q}_2 &= BR_2(q_1) - q_2 \end{aligned}$$

   Where $BR_1(q_2)$ stands for the amount of q_1 that is optimal for firm 1 given that firm 2 produces q_2. Suppose that c_1 and c_2 are such that only one firm is producing in Nash Equilibrium. Draw the phase portrait of this system and find the explicit solution. Determine the stability properties of the steady state.

3. (25 points) Find the explicit solution of the following system of linear differential equations:

   $$\begin{aligned} \dot{y}_1 &= y_1 - 5y_2 \\ \dot{y}_2 &= 2y_1 - 5y_2 \end{aligned}$$

   Determine the stability properties of the steady state $y_1 = 0;\ y_2 = 0$.