

4113: Math Econ Midterm, DUE THURSDAY MARCH 8 IN CLASS

1. Solve

$$\begin{aligned} \max_{x,y,z \in \mathbb{R}} \quad & (x+y)^2 + 5xy + x - y \\ \text{s.t.} \quad & x^2 + y^2 \leq z^2 \\ & z \leq 1 \\ & x^2 - \frac{1}{4} \leq y \\ & x, y, z \geq 0 \end{aligned}$$

It is enough to give an equation in one variable (say y) and say that the solution to this equation is the optimal y . Also specify how to find optimal x and z in terms of that optimal y .

2. For each value of the parameter $q \in [-1, 1]$ solve the following problem and find dV/dq

$$\begin{aligned} V(q) = \max_{p \in \mathbb{R}} \quad & qp^2 - p \\ \text{s.t.} \quad & 0 \leq p \leq 1 \end{aligned}$$

3.

$$\begin{aligned} \max_{x,y \in \mathbb{R}} \quad & -x^2 - 2y^2 - xy - 2x \\ \text{s.t.} \quad & x^2 + y^2 \leq 4 \\ & x + y \leq 2 \\ & x, y \geq -2 \end{aligned}$$

4.

$$\begin{aligned} \max_{x,y \in \mathbb{R}} \quad & x \\ \text{s.t.} \quad & y - x^4 \leq 0 \\ & x^3 - y \leq 0 \\ & x \leq \frac{1}{2} \end{aligned}$$

5. Consider the problem of maximizing the utility of a consumer in an exchange economy, keeping the utility of the other consumer no less than some fixed level \bar{u}_2 :

$$\begin{aligned} \max_{x_1, y_1, x_2, y_2 \in \mathbb{R}} \quad & u_1(x_1, y_1) \\ \text{s.t.} \quad & u_2(x_2, y_2) \geq \bar{u}_2 \\ & x_1 + x_2 \leq w_x \\ & y_1 + y_2 \leq w_y \\ & x_1, y_1, x_2, y_2 \geq 0 \end{aligned}$$

Suppose that u_1 and u_2 are strictly increasing in all arguments. Show that at the solution point $(x_1^*, y_1^*, x_2^*, y_2^*)$ the first three constraints bind. Suppose for simplicity that $x_1^*, y_1^*, x_2^*, y_2^* > 0$. Find the conditions that $x_1^*, y_1^*, x_2^*, y_2^*$ should satisfy.