The Multiplier with imports

We assume that countries are spending fixed % of their GDP on buying goods in other countries, e.g. on imports.

Example. Suppose
C = 200 + 0.5DI
T = 100
Tr = 0
G = 100
I = 200
IM = 0.2Y
X =300

Here IM = 0.2Y, which implies that 20% of GDP are spent on imports. Remember that DI = Y – T + Tr = Y – 100. Equilibrium on the demand side Y* solves the following equation

$$Y = C + I + G + X - IM = 200 + 0.5(Y - 100) + 200 + 100 + 300 - 0.2Y$$

Rearranging, we find

$$Y^* = (200 - 50 + 200 + 100 + 300) / (1 - (0.5 - 0.2)) = 1071$$

In general, Marginal Propensity to Import (MPI) is % of extra $1 of income that is spent on imports, e.g. suppose MPC = 0.9 and MPI = 0.1. This implies that
C = 0.9DI + *some constant* and IM = 0.1Y. Out of each new dollar of income, $0.90 is spent, but of that $0.90, $0.10 is spent on foreign goods.

So, Marginal Propensity to Consume Domestic Products
MPC domestic = MPC - MPI
= 0.90 - 0.10
= 0.8
only $ .80 of an extra $1 in income is spent on domestic goods.

MPC domestic is the number we need to accurately calculate the multiplier

Multiplier with imports = 1 / (1 – MPC domestic) = 1 / (1 - (MPC – MPI))

in our example, with MPI = .1:
Multiplier with imports = 1 / (1- (.9 - .1)) = 1 / (1- .8) = 1 / 0.2 = 5

Example from hw3

Suppose there are only two countries: the US and Mexico. Consider the following data:
MPC_{MX} = 0.4 (MPC in Mexico)
MPI_{MX} = 0.03 (so 3% of an additional $1 of income in Mexico is spent on the American goods)

MPC_{US} = 0.6 (MPC in the US)
MPI_{US} = 0.1 (so 10% of an additional $1 of income in the US is spent on the Mexican goods)

a) *Suppose Mexican government increases government spending by $1 billion. By how much does Mexican GDP grow?*

   To solve this we use the Multiplier with imports.
   $M_{MX} = 1/(1 - (MPC_{MX} - MPI_{MX})) = 1/(1-(0.4 – 0.03)) = 1.587$

   So in Mexico $\Delta Y^* = M_{MX} \cdot \Delta G = 1.587 \cdot \$1$ billion = $1.587 billion

b) *Calculate the increase in the American GDP induced by the increase in Mexican GDP.*

   As we said above, Mexico spends $MPI_{MX} = 0.03$ of any extra $1 income on foreign goods (in our example on American goods). So out of $1.587 billion increase in Mexican GDP, $\Delta X_{US} = MPI_{MX} \cdot \$1.587$ billion = $0.03 \cdot \$1.587$ = $47.61 million will be spent in the US. This increase of spending in the US will start the multiplier effect. However, Americans also spend some fixe % of GDP on bying foreign goods (in our example, Mexican goods). Therefore, multiplier with imports in the US is

   $M_{US} = 1/(1 - (MPC_{US} - MPI_{US})) = 1/(1-(0.6 – 0.1)) = 2$

   So in the US $\Delta Y^* = M_{US} \cdot \Delta X_{US} = 2 \cdot \$47.61$ million = $95.22 million

c) *Calculate the amount by which Mexican GDP grows as a result of the increase in the US GDP.*

   We do exactly the same calculation as in b), only the role of two countries is reversed. US spends $MPI_{US} = 0.1$ of any extra $1 income on foreign goods (in our example on Mexican goods). So out of $95.22 million increase in American GDP, $\Delta X_{MX} = MPI_{US} \cdot \$95.22$ million = $0.1 \cdot \$95.22$ million = $9.522 million will be spent in Mexico. This increase of spending in Mexico will start the multiplier effect. Remember that we have to use Mexican multiplier with imports now, calculated in a)
   So in Mexico $\Delta Y^{**} = M_{MX} \cdot \Delta X_{MX} = 1.587 \cdot \$9.522$ million = $15.11 million

The Multiplier with taxes

Usually we assume that taxes are fixed. However, in reality most taxes depend on the income of people and firms. We can think of it as dependence of tax on GDP.

Suppose we have the following data:
$C = 5 + 0.8 \cdot DI$
$I = 15$

$G = 20$
$NX = X - IM = 5$
$Tr = 0$
$T = 0.1 \cdot Y$ (i.e. taxes are now variable; = 10% of income)

We want to find the Equilibrium on the Demand Side. We should solve the equation

$$Y = C + I + G + NX$$

We know that $DI = Y – T + Tr$. So we rewrite the equation above as

$$Y = 5 + 0.8 \cdot (Y – 0.1 \cdot Y + 0) + 15 + 20 + 5$$

From this we get $Y^* = (5+15+20+5) / (1 – 0.8 \cdot (1 – 0.1)) = 160.71$

Now we want to find the multiplier with taxes in general. Suppose that
$C = c + MPC \cdot DI$
I is constant
G is constant
$NX = X - IM$ is constant
$Tr = 0$
$T = tY$

Here again, $DI = Y – T + Tr = Y - tY + 0$. Also c is some constant number and $t \cdot 100\%$ is the size of the income tax. Equilibrium on the demand side is Y^* which solves

$$Y = C+I+G+NX = c + MPC \cdot (Y - tY) + I + G + NX$$

So, by rearranging the terms we get

$$Y^* = (c + I + G + NX) / (1 - MPC \cdot (1-t))$$

This implies that any change in I, G or NX, call it Δ, will change Y^* by

$$\Delta Y^* = \text{Multiplier with taxes} \cdot \Delta$$

Where

$$\text{Multiplier with taxes} = 1 / (1 - MPC \cdot (1-t))$$

Example. Suppose
$C = 10 + 0.7 \cdot DI$
$I = 25$
$G = 15$
$NX = X – IM = 10$
$Tr = 0$
$T = 0.2Y$

Suppose also that NX goes up by 10. So $\Delta NX = 10$. By how much will Y^* change? You see that in this example we have $T = 0.2Y$. So we should use the multiplier with taxes. In the example it is

$$M_{taxes} = 1/(1 - 0.7(1-0.2)) = 2.27$$

Therefore,

$$\Delta Y^* = 2.27 \cdot \Delta NX = 22.7$$